Scenario Analysis in US Army Decision Making

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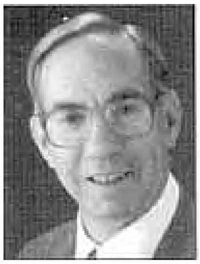

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Abstract

This article describes *scenario analysis,* a technique for modeling and optimizing decisions under uncertainty. The description is informal and is oriented toward working analysts rather than theoreticians. It includes a description of the successful implementation of this method by the US Army TRADOC Analysis Center, White Sands Missile Range, NM, in a decision support system that has had extensive use in Army analy-

Introduction

Scenario analysis is a method for optimizing under uncertainty, in which the possible "states of the world" are represented by a finite number of scenarios, each having a fixed, known probability.

As we explain later, by appropriate reformulation this problem can be reduced to a large deterministic optimization problem, which can be solved by efficient implementations of Linear Programming (LP) such as are found in optimization modeling languages. The technique uses basic ideas common in stochastic programming, which we explain informally below. Formal proposals for such methods appeared in the work of Rockafellar and Wets and Robinson.[1,2]

The basic reason for using a method like scenario analysis is that it allows us to compute hedged decisions. Under uncertainty the best overall decision might not be best for any individual scenario, so we have to look at all scenarios simultaneously to optimize. In many operational situations a hedged approach is essential for making good decisions. An example is military force design. A force may have to be employed in many different environments, and it must not fail in any of these. However, there may not be enough resources available to build a force that will perform brilliantly in each. Therefore we try to design a force that will do fairly well in all scenarios, as well as can be done with the resources at our disposal. This methodology also lets us make tradeoffs within the hedged decision paradigm, considering such questions as:

- How much performance can we afford?
- If we give up some lethality, can we gain survivability?
- What is the tradeoff between cost and survivability at a given level of lethality?

Analysis of this kind has numerous applications. We discuss military analysis below, emphasizing in particular the extensive use of scenario analysis by the Army. Nonmilitary application areas include finance (multistage investment models with scenarios) and policy analysis. Extended versions can also be applied in quality improvement contexts such as design centering.

The following section explains the need for a method like scenario analysis to optimize in an uncertain environment. After that, we describe the modeling technique. Finally, we discuss how the method has been successfully implemented by the US Army TRADOC Analysis Center, White Sands Missile Range (TRAC-WSMR), and has been used in a number of Army studies.

Why use scenario analysis?

We begin with a standard problem of optimization over time: we are given time periods $1,\ldots,T$ and in each time period we have to make certain decisions. Some measure of cost or benefit is given, and this is to be optimized with respect to the decisions available in the various periods. Of course, decisions taken in earlier periods may affect those available in later periods.

A familiar example of this kind of situation is a time-staged linear programming problem. The decisions at each stage are modeled by a linear program, and the constraints of these are (often loosely) connected by the interdependence of decisions in different time periods. This kind of problem is well understood, though not always easy to solve.

Now consider a variant of this situation, in which during the first time period any one of a finite number of different alternative situations may occur (each with a fixed, known probability). The portion of the optimization problem (linear program, for example) representing the first-stage decisions will be *different* for each of these alternatives. For each of the first-stage alternatives, there is then a finite set of alternatives that may happen in the second period, and so on. If we start at the beginning and go through a particular alternative at the first stage, a particular alternative at the second, and so on through all T stages, we obtain a single realization, or sample path, of the random process just described. Such a realization is called a *scenario.*

In this probabilistic model, each scenario is a time-staged optimization problem of the sort originally described. However, now there are many of these scenarios, and of course we don't know in advance which will occur. This introduction of multiple scenario situations immediately poses a problem: how are we to combine the performance measures of different scenarios? That is, how are we to deal with the fact that, for example, a certain set of decisions might perform very well against one group of scenarios, but poorly against another?

In this article we'll assume that the use of expected performance (in the probabilistic sense) is satisfactory. That is, we will accept as a measure of performance the expected value, or average, of the performances against different scenarios, when the probabilities assigned to those scenarios are taken into account. This is not the only measure that could be used, but it is probably the simplest to deal with, and it fits current practice in many areas.

Note that this use of an expected value performance measure is not at all the same thing as the common use of "expected-value models" in which a single run, essentially deterministic, simulation is made in which stochastic elements are individually and systematically replaced by their expected values. That procedure is invalid as a method for modeling anything, since the outcome cannot be reliably related to the average of the outcomes under the individual scenarios, or to any other quantity of interest. Rather, the expected value performance measure that we are using corresponds to use of a Monte Carlo simulation process, but (as we see below) with a certain degree of increased structure.

A well-known difficulty of Monte Carlo simulation is the large number of individual runs that need to be made in order to take into account adequately the variation in many different parameters. This problem of dimensionality is compounded if in the process of simulation we also wish to optimize, as we are assuming here. Even moderate numbers of variables and modest amounts of variation can then lead to enormous amounts of computing.

It is this problem of dimensionality that the technique of scenario analysis was designed to overcome. In essence, it does this by employing finite distributions (perhaps approximations or estimates of the actual distributions, if the latter are continuous), and by clever organization of the sources of variation so that the overall problem can be solved using large-scale mathematical programming methods. Thus, scenario analysis has the potential to contribute significantly in many applications in which Monte Carlo type analysis is desired but in which dimensionality is a problem.

Overview Of Scenario Analysis

This section establishes notation and explains the general procedure of scenario analysis. To begin with, we assume that the uncertainty in the model can be adequately described by a finite (possibly large) set of scenarios. These scenarios are to be understood as descriptions of the environment. They incorporate those things that cannot be changed by the decisions made in the course of the optimization, whereas the things that can be changed are modeled as part of the optimization problem.

Each scenario is understood to evolve over a fixed (finite) number of time periods. This number of time periods is the same for all scenarios. Within each time period the actors can make certain decisions. We index the scenarios by the letter s (running from 1 up to S), the time periods by the letter t (running from 1 up to T), and the decisions made in scenario s at time t by the vector x_{st}, whose dimensionality could depend on both s and t. The collection of vectors $x_{s1}, \ldots, x_{sT}$ will be denoted by x_s, and we interpret it as a larger vector. This is the complete sequence of decisions made in the single scenario s, for time periods 1 up to T.

Each scenario is given a fixed, positive probability p_s of occurrence, and these p_s sum to 1. We assume that they are fixed at the beginning of the analysis, and are known to the decision makers.

We also assume that once the decisions x_s have been made, there is a measure of overall cost or loss, given by $f(s, x_s)$; the first index s is used to indicate that the cost measure might well be different in different scenarios. Of course, one might as well use a measure of gain or merit if desired, and this would just be the negative of f. In the formulation implemented by TRAC-WSMR the functions f are linear in x, though the methodology can accommodate more general expressions.

$$\sum_{s=1}^{S} p_s f(s, x_s),$$

The overall expected cost, given the decisions x_s in each of the scenarios, will be

representing the cost incurred in each scenario weighted by the probability that the scenario will occur. Therefore this overall cost is an expected, or average, cost given the decisions made. If we denote the collection of all scenario decisions x_s by the vector x, then we can write the overall cost as $f(x)$, where this is to be understood as the weighted sum just described.

Finally, the decision x_s to be made in scenario s is not completely arbitrary; we suppose that there is a closed, bounded convex set C_s in which x_s has to lie. Decisions x_s outside of this set are not allowable; decisions in it are called *feasible*. In the linear programming case, C_s is represented by a finite collection of linear equations and linear inequalities.

With this background, we can make a first try at describing the problem that the decision maker faces. This would say that we try to choose the *policy* x (that is, the entire collection of decisions x_{st} for s running from 1 to S and t running from 1 to T), in such a way that x_s belongs to C_s for each s and, among all such feasible decisions, the value of $f(x)$ is least. In words: choose actions that are feasible and that yield the least expected cost among all possible feasible actions.

This description is still incomplete, though, because if we are making a decision in scenario s at time 1 we cannot expect to know all of the information that will be disclosed to us as the scenario evolves through times 2, ..., T. We deal with this by allowing "branching" of the scenarios as time evolves. However, this branching introduces extra complexity into the problem, because we must be careful not to use future information to make present decisions.

For an artificial example, suppose that we have two time stages and at the first stage a coin flip determines what the second-stage environment will be. We could express this by setting up two scenarios, one for the case in which the coin flip turned out Heads, and the other for Tails. However, we would have to introduce constraints to ensure that each decision variable in these two scenarios had the *same* value in the first time stage. The reason for this is that at the first stage we have not yet flipped the coin, so we don't have the information needed to know which scenario we're in. As we can't use information we don't yet have, we have to require the first-stage variables to have the same values.

If a policy satisfies this information constraint, we call it *implementable*. This requirement of implementability leads to a complex system of constraints tying together decisions in different scenarios at the same time period. However, we can express the problem clearly in words by saying that we want to choose a policy that yields the least expected cost (or the greatest expected benefit, if we are maximizing) among all policies that are both feasible

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and implementable.

Fortunately, if the number of time stages is not too large, then with the aid of modern optimization tools such as modeling languages one can often model and solve such problems as large-scale linear programming problems by using a good commercial LP solver. This is the approach followed by TRAC-WSMR, using the GAMS modeling language with the CPLEX solver. For example, one early brigade model analyzed by TRAC had approximately 3,800 constraints (not counting bounds) and 3,300 variables; this problem is readily manageable with GAMS.

Implementation and Use by TRAC-WSMR

TRAC-WSMR has implemented this methodology as part of a sophisticated analysis capability to develop optimum policies for design of forces and associated equipment based on specific Army requirements. For a description of some of the implementation considerations, see "Scenario Analysis for Combat Systems" by Laferriere.[3] The capabilities of the TRAC-WSMR analysis methodology include:

- Determining families of combat-effective systems.
- Providing means to conduct comparative analyses of force structure.
- Identifying resources needed to staff and maintain the resulting forces.

The methodology consists of embedding the basic mathematical technique of scenario analysis in a sophisticated decision support system. At the "front end" of the system, exploratory data analyses are run, using results from high-resolution combat models such as CASTFOREM. These analyses provide the expected values for several measures of effectiveness for each alternative from each of the scenarios. They provide the means for grouping like-capability systems, and identify significant contributors to the force by scenario. They also provide our first look at how well the different alternatives enhance the base case force's capabilities within a specific scenario, and provide the initial values for the coefficients used in the follow-on scenario analysis model.

The scenario analysis optimization then provides families of combat effective systems, identifies competing high valued systems, and provides alternative families of systems and unit costs. It does so by using a linear programming model to select the mix of combat systems that gives the best improvement over the combat effectiveness of the base case force across several combat scenarios. The LP model is thus a system integration tool used to identify families of combat systems and their expected effectiveness.

To ensure a robust, effective and flexible force that can handle worldwide contingencies, the model employs uncertainty in two ways. First, uncertainty is represented by identifying the possible events that the force is expected to handle. These events could be single battles, or a series of multiple battles. The latter would lead to a more robust and flexible force capable of handling more than one battle, battle type and geographical location throughout the world. For past studies, the developed forces were expected to fight three battles, developed from combinations of the several scenarios. Second, a probability is assigned to each event identified. These probabilities reflect the decision maker's estimate of the importance or potential of each scenario. In past studies, scenarios have been assigned equal probability; however, excursions were conducted to determine the impact of a high (or low) probability on the force mixes and their effectiveness.

Finally, a "back end" decision support system provides lists of alternatives based on combat capability, and facilitates presentation of results to decision makers through visualization devices such as Pareto (efficient frontier) diagrams based on combat effectiveness and cost.

Figure 1 displays the results of a notional study. The left-most column lists the major alternatives of the study, while across the top are the leading alternative candidates and Best Families (BF) developed through the scenario analysis methodology. The chart identifies which new systems are included in each force, and their expected effectiveness. For this study, effectiveness parameters deemed important were threat losses, threat armor losses, threat artillery losses and Blue personnel losses. In addition to these, past studies have employed Blue armor losses, Blue artillery losses and transportation flow (planeloads required to transport the force) as effectiveness parameters.

Figure 2 demonstrates how notional alternatives would rank, based on improvements over the base case, and reflecting the consequences of alternative weightings on each of the lethality and survivability MOEs. Across the top of the chart are the weighting schemes for the MOEs.

The first two columns list the alternatives and their scores when lethality is

Scenario Analysis
Major System Mixes

	Leading Alternatives				Families of Systems				
System	BC	A1	A2	A3	BF1	BF2	BF3	BF4	BF5
ALT 1		X			X	X	X	X	
ALT 2			X		X	X			
ALT 3				X	X	X	X	X	X
ALT 4					X	X	X	X	X
ALT 5					X	X	X	X	X
ALT 6					X	X	X	X	X
ALT 7					X	X	X	X	
ALT 8					X		X		
Total Threat Losses	2184	2311	2306	2214	2508	2508	2454	2457	2497
Threat Armor Killed	657	735	723	686	810	805	800	800	790
Threat Arty Killed	110	150	225	210	235	230	230	230	210
Blue Pers Losses	428	388	347	379	323	329	323	329	371

Notional

BF1 - Best Family Mix Model
BF2 - BF1 Less Most Expensive
BF3 - BF1 Less 2nd Most Expensive
BF4 - BF1 Less both Most Expensive
BF5 - High Performance At Lowest Cost

Figure 1. Display of results for notional study

weighted at 100% and survivability at 0% (labeled 100-0), and this display continues to the last two columns where the alternatives are evaluated purely on Blue survivability (0-100). For each weighting scheme the alternatives are listed in ascending score order. The scores represent expected improvement over the base case for threat losses and survivability when combined according to the weighting scheme employed. Most decision makers would evaluate alternatives by considering some combination of lethality and survivability. Thus, the central 6 columns in the chart provide an indication of how well an alternative would rank under differing levels of importance for lethality and survivability.

If for each alternative we plot performance improvement over the base case (good) against total Army cost (bad), we can develop a list of systems and mixes of systems that provide the most benefit for a given level of cost, or equivalently the least cost for a given level of benefit. These selected mixes are *Pareto optimal* for the two measures of improvement and cost. This plot gives us a geometric visualization of the *efficient frontier,* comprising all of the Pareto optimal mixes.

Figure 3 illustrates the outcome of this analysis. It shows the efficient frontier as a solid line, passing through the Pareto optimal mixes. For those alternatives not on the efficient frontier, there exists an alternative that provides more improvement over the base case at a reduced cost.

Figure 3 illustrates a very important point about presentation of this kind of analysis to a decision maker. A quick glance at the Pareto diagram shows that the mix designated by BF5 gives substantially better performance than any of the alternatives designated ALT1 – ALT8, and at lower cost than most. The other "best family" mixes, BF1 – BF4, all give better performance than BF5, but by negligible amounts and at substantially greater cost. The diagram therefore allows a decision maker to see graphically the tradeoffs between cost and effectiveness involved in comparing the different mixes. Although one could certainly present this information in tabular form, the diagram makes it much easier to comprehend. In this case, assuming that the decision maker was comfortable with the 75 – 25 weighting, he or she would very likely want to single out mix BF5 as a possible best choice. It is

Rankings of Alternatives

Notional

← Lethality - Survivability →

100-0	score	75-25	score	50-50	score	25-75	score	0-100	score
ALT 1	20	ALT 2	17	ALT 2	18	ALT 2	18	ALT 2	19
ALT 2	17	ALT 1	17	ALT 1	15	ALT 1	12	ALT 7	12
ALT 3	17	ALT 3	14	ALT 3	11	ALT 5	12	ALT 5	12
ALT 4	14	ALT 4	12	ALT 5	11	ALT 7	11	ALT 1	10
ALT 5	10	ALT 5	11	ALT 4	10	ALT 3	9	ALT 4	6
ALT 6	8	ALT 7	8	ALT 7	9	ALT 4	8	ALT 3	6
ALT 7	7	ALT 6	7	ALT 6	4	ALT 6	2	ALT 6	0
ALT 8	2	ALT 8	1	ALT 8	1	ALT 8	1	ALT 8	0

Figure 2. Ranking of alternatives for notional study

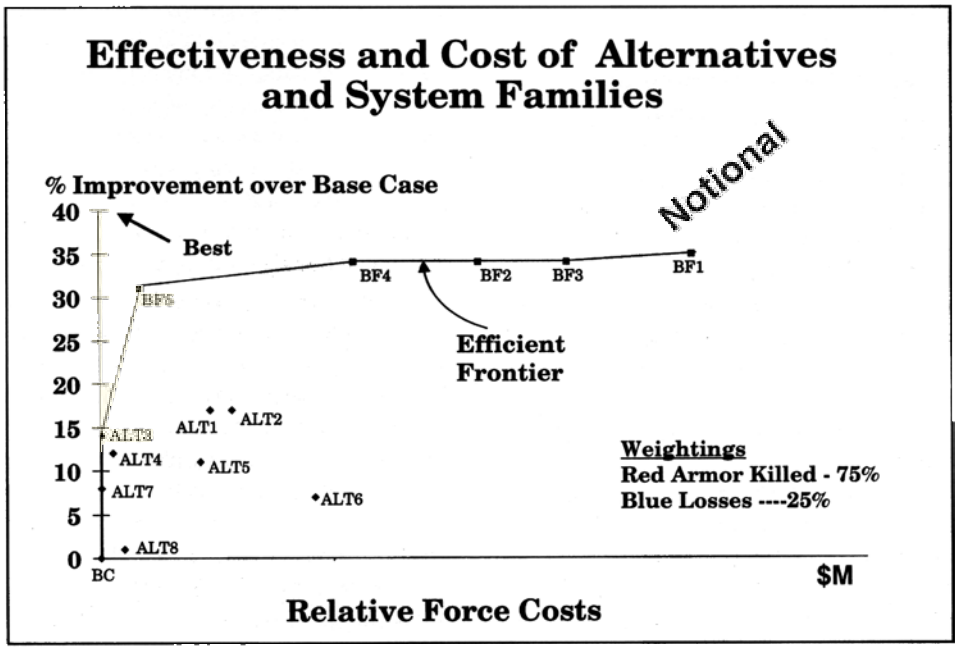

Figure 3. Efficient frontier for 75-25 weighting

also worth noting that the initial list of alternatives did not include this mix: the scenario analysis methodology identified it as a candidate for examination. This illustrates one of the strengths of the optimization methodology we have presented here.

As we have seen, by combining the scenario analysis methodology with the presentation methods just described we obtain a quick and flexible tool with which a decision maker can re-evaluate current decisions against new information (altered planning horizons, changes in priorities, new combat tactics, doctrine, systems, or new threat capabilities). The model can suggest new decisions based on the changed information, and in some cases it has been possible to provide real-time response in face-to-face conferences with senior decision makers.

Here is a selection of studies in which this methodology has been used over the last 10 years:

- An Armor Anti-Armor Mix Methodology – 1990

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GIPS

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Like most manufacturing processes, NIMA follows established process flows which possess variable production times and costs. However, within GI the largest source of variation is the complexity of each job (i.e., the difference in mapping Kansas farmland vs. Southern California). Neither of these variation sources is modeled, since deterministic production standards based on averages represented the best available planning data when GIPS was created. Increased experience with modeling and simulation is expected to show the importance of variation management to productivity, drive collection of better production metrics, and ultimately produce stronger and even more useful modeling results.

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Author Biography

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- The ASM and the TSIP COEAs – 1991
- LM VII Artillery Ammunition Mix Study – 1991
- Scenario Analysis for Combat Systems – 1992 *
- Early Entry Analysis: Division Ready Brigade – 1994 *
- Analysis of Amphibious Assault Fire Support Requirements – 1995
- Antiarmor Resource Requirements Study – 1996 *
- Techniques for Increasing Efficiency and Accuracy of Data for Mix Analysis – 1998
- Marine Corps Antiarmor Study – 1999

The studies marked with asterisks in the above list won the Dr Wilbur B. Payne Memorial Award for Excellence in Analysis, presented by the Deputy Under Secretary of the Army (Operations Research).

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Biography

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